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APOGEE RAISING TECHNIQUE FOR THE MAGNETOSPHERIC MULTISCALE FORMATION FLYING MISSION

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The NASA Goddard Space Flight Center's Magnetospheric Multiscale (MMS) program involves a four-spacecraft tetrahedral formation flying mission intended for launch in 2014. The mission's high Earth orbits are designed to provide repeated excursions through the magnetosphere and magnetotail for measurement of interaction phenomena between the solar wind and magnetosphere, including magnetic reconnection events. The first of two main science mission phases requires a 1.2 by 12 Earth radii (Re) orbit, while the second phase requires a 1.2 by 25 Re orbit. A transition between the two science phases is for independently raising the apogees of the four spacecraft to 25 Re in stages, followed by re-initialization of the tetrahedral formation. A variety of stringent operational requirements and constraints, plus design features of these spinning spacecraft, pose significant challenges to the apogee raising design. This paper has two main parts. The first part focuses on the strategy and methodology for, and the solutions to, the apogee raising design problem. The second part presents and discusses nominal solutions to the problem of recovery from off-nominal finite burns and maneuver contingency scenarios.

INTRODUCTION

The Magnetospheric MultiScale (MMS) Mission consists of four identical spacecraft flying in formation in highly eccentric Earth orbits.[1] Current plans call for these octagonal spin-stabilized spacecraft (see Figure 1) to be launched together from the Kennedy Space Center aboard an Atlas-Centaur vehicle in 2014. The launch vehicle will insert the four stacked spacecraft into an orbit of 185 km by 12 Earth radii (Re). The mission timeline is divided into two main science phases.[1] The first phase requires the MMS spacecraft to maintain a formation in the shape of a tetrahedron positioned around a reference orbit of dimensions 1.2 × 12 Re and inclined 28.5 deg to the Equator. The second phase increases the apoapsis radius to 25 Re with the return of all 4 spacecraft to a loose formation at apogee, where a tetrahedral formation can be reestablished for the beginning of the science portion of the phase. In Keplerian terms, the raising of apogee from 12 to 25 Re requires an impulsive ΔV of approximately 238 m/sec per spacecraft. There are significant challenges associated with the MMS apogee raising sequence. One such challenge will be the execution of multiple burns over several perigee passages using 4 lbf radial thrusters in pulsed mode, onboard spacecraft spinning at 3.0 rpm[1]. Other challenges posed by various operational constraints include no simultaneous maneuvers among the spacecraft, maintaining a minimum interval of 24 hours between any two maneuvers, and following a maneuver a spacecraft must not maneuver during its next perigee passage to allow an unperturbed tracking arc for orbit determination. Additionally, there is a limit on the time available to complete the apogee raising sequence to reach the science area of interest at a specific time.

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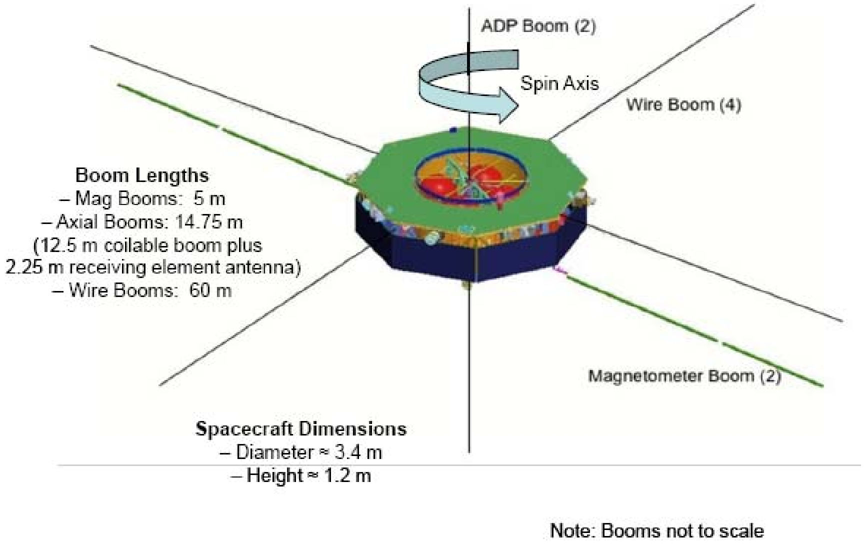

Figure 1. Deployed MMS Spacecraft are Spin-stabilized at 3.0 rpm

The apogee raising technique developed consists of two parts. First, a desired number of perigee maneuvers to reach the 25 Re apogee is determined. It is important that this number be an even number because the approach depends on pairings of maneuvers, where each maneuver in a pair is referred to as a leg. Selection of this number is, at least in part, arbitrary, but as will be demonstrated later in the paper, eight such maneuvers are reasonable for MMS. The second part of the technique involves computation of the size of each of the maneuvers broken down further in the following way. First, an even number of equal decrements of mean motion are computed, based on dividing the synodic mean motion between the initial (12 Re) and final (25 Re) orbits by half the number of intended maneuvers. The desired mean motion decrement goals are then converted to orbit period goals for the purpose of ΔV targeting. The next step involves dividing each period goal in half, with each half achieved by a separate perigee maneuver. With the last step the full complement of desired maneuver pairs is specified. The initial maneuvers will be conducted one spacecraft at a time, approximately one perigee passage apart by turns. The spacecraft will then execute their second set of burns in precisely the reverse order, again one perigee passage apart, with the result that, after all the second burns are complete, all four spacecraft will return to apogee together in phase with one another. The burns for achieving the remaining decremental mean motion changes are conducted in the same 1-2-3-4-4-3-2-1 manner.

At the end of the apogee raising campaign, all four spacecraft are re-phased, i.e., they all return to apogee together in a ‘string of pearls’ configuration; this is for safety reasons and to position them for re-initialization of the formation. One of the advantages of the technique is that opportunities for re-formation are provided at intermediate stages along the way in the event that spacecraft contingencies prevent completion of the ascent to 25 Re. Other advantages result from the technique’s division of the synodic mean motion into equal decrements. These include the fact that during the campaign more orbital revolutions occur in smaller, shorter period orbits than in longer period orbits, which helps in meeting the time limit constraint. Another advantage is that the early ΔVs are the smallest, which reduces the duration and gravity losses associated with the finite burns, thereby saving fuel and easing operations.

The paper also addresses techniques and mitigation strategies for recovery from off-nominal burns (errors within 3σ) and contingency burns (i.e., errors significantly larger than 3σ, including missed burns) using the apogee raising technique described above. The handling of these off-nominal and contingency scenarios involves what are termed ‘short term’ and ‘long term’ recoveries. Short term recoveries are defined as those where the spacecraft can be recovered within nine revolutions following the anomaly or contingency. For long term recoveries, it is assumed that the anomalous spacecraft is unable to recover quickly yet the other spacecraft must meanwhile continue their nominal ascent maneuvers. In such cases, the "stranded" spacecraft must complete its apogee raising and re-phasing with the rest of the formation at a later time.

BACKGROUND ON THE MMS APOGEE RAISING PROBLEM

Launch of the MMS spacecraft is currently planned for August 2014. An Atlas-Centaur V launch vehicle will inject the stack of four identical 1275-kg spacecraft into an 1.03 Re (6378 + 185 km) by 12 Re (76,538 km) orbit, inclined approximately 28.5 degrees to the Equator. Following a series of maneuvers to raise perigee to 1.2 Re, the orbit period is about 0.98 day. The mission's main phases are described in References 1 and 2. The tetrahedral formation is initialized by the start of science in Phase 1, then maneuvered to achieve and maintain various tetrahedral volumes (side-length spatial scales from tens to hundreds of kilometers) throughout the regions of interest during the science collection phases.[3] The science mission is planned to span two years. Phase 2a includes the apogee raising campaign, which is the focus of this paper, and formation re-initialization with mutual separation of 400 km to start Phase 2b science. The spacecraft are not required to be in a tetrahedron during the apogee raising. The tetrahedron will be re-formed after all four spacecraft are established in the 1.2 by 25 Re orbit. The apogee raising increases the orbit periods to 2.78 days.

Apogee Raising Requires Multiple Burns

To begin Phase 2, the MMS spacecraft are required to raise their initial 1.2 Re by 12 Re science orbits to final orbits measuring 1.2 by 25 Re (159,454 km). This represents an increase of apogee height of 13 Re (82,916 km) and an increase in the orbital period of 1.80 days. Given that the perigee velocity of the 12 Re orbit is 9.731 km/sec, and for the 25 Re final orbit it is 9.969 km/sec, the required impulsive ΔV is approximately 238 m/sec.

The spacecraft design and mode of operation precludes imparting a ΔV of 238 m/sec with a single maneuver. The MMS spacecraft performs maneuvers with radial thrusters operated in pulsed mode while spinning at 3.0 rpm. The spacecraft are equipped with two groups of four 4-lbf radial thrusters, with each group mounted on diametrically opposite faces of the octagonal bus.[*] Every half-spin one radial thruster group will fire, such that both groups will have fired once every full spin. Each group generates approximately 71 N of force (F) with a specific impulse (Isp) of approximately 224 sec. This means that—assuming constant force and specific impulse throughout the burn, with gravitational acceleration g = 9.8 m/sec^2—the fuel flow rate, $\dot{m} = F/gIsp$, is approximately 0.0323 kg/sec.

With an anticipated pulse width of 64 deg, and the spin period of 20 sec, then a single pulse takes 3.56 sec and the summed firing time δt of the two groups amounts to 7.11 sec every full spin. The propellant used per spin would then be $m_P = \dot{m}\,\delta t$, or 0.2297 kg, and the total impulse delivered every full spin would be $I_T = m_P\, g\, Isp$, or 504.2 N-sec. Assuming a spacecraft mass of approximately 1,200 kg at the beginning of the apogee raising phase, at best the ΔV imparted every spin will be approximately 0.42 m/sec. Three spins per minute equals 1.26 m/sec per minute; therefore, it requires a burn totaling 238 m/sec ÷ 1.26 m/sec per min = 189 min, or 3.15 hours, to consume the propellant equivalent to 238 m/sec. Given that traversal through the perigee region is very rapid for these orbits (for a 1 by 12 Re orbit, flight time between the true anomalies of 270 and 90 deg is approximately 65 min), even a rough calculation[†] such as this one shows

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[*] Typical of spinners, the radial thrusters are mounted on the side panels of the octagonal bus and aligned normal to the spin axis, whereas the top and bottom deck 1-lbf axial thrusters are aligned parallel to the spin axis. See Reference 1 for more detail on the MMS propulsion system.

[†] Factors ignored in this rough calculation include a monopropellant blowdown system's behavior where the magnitude of thrust and *Isp* decrease as fuel pressure declines, the spin-arc losses deriving from the 64 deg pulsewidth (about 5% for MMS), and the finite burn arc gravity losses which increase with the length of the burn. Another major factor is the MMS peculiarity that—due to the power and science-mandated spin axis attitude—the spin plane will in general be inclined to the orbit plane, which results in losses from imparting a ΔV out-of-plane component.

that it is clearly infeasible for MMS to raise apogee to 25 Re with a single burn; therefore, multiple burns will be required.

Apogee Raising Requirements and Constraints

The development of the apogee raising technique took into account a variety of mission and spacecraft requirements, guidelines, constraints and limitations derived from the needs and realities of the science, spacecraft design, propulsion system characteristics, tracking and navigation requirements, and the ground system and operations. However it is important to emphasize that there is no requirement to maintain formation flying during the apogee raising. Hence, the spacecraft are free to make their ascent independently and may therefore at times separate from each other significantly.

The following constraints and guidelines were considered when developing the apogee raising technique:

- Phase 2a begins at the epoch where the 1.2 by 12 Re orbit apogee vector has rotated to 1000 hr in the Geocentric Solar Ecliptic (GSE)[1] rotating frame of reference (see Figure 2).
- Each spacecraft is required to have at least one open no-burn perigee passage between its orbit maneuvers to allow for the orbit determination.
- A minimum interval of 24 hours is desired between any two orbit maneuvers (any spacecraft) so as to provide adequate time for operations to calibrate the previous maneuver and plan the next maneuver.
- At the end of the apogee raising campaign the four spacecraft must be close enough to each other in the apogee region so as to minimize the maneuvers and ΔV needed to reform the 400-km side-length tetrahedron desired for the Phase 2b start. However, they must not be so close as to create collision hazards.
- The spacecraft must complete the apogee raising campaign and be re-initialized in its new tetrahedral formation prior to the start of Phase 2b, which is defined as the epoch when the apogee vector has rotated to the point in the GSE frame where its x-axis coordinate has reached the value of −10 Re (see Figure 2). This effectively limits the time available to complete the apogee raising. This time may vary somewhat depending on the particular orientation of the orbit with respect to the ecliptic plane, which varies with launch window case, but is generally considered to be about 11 weeks on average.[2]

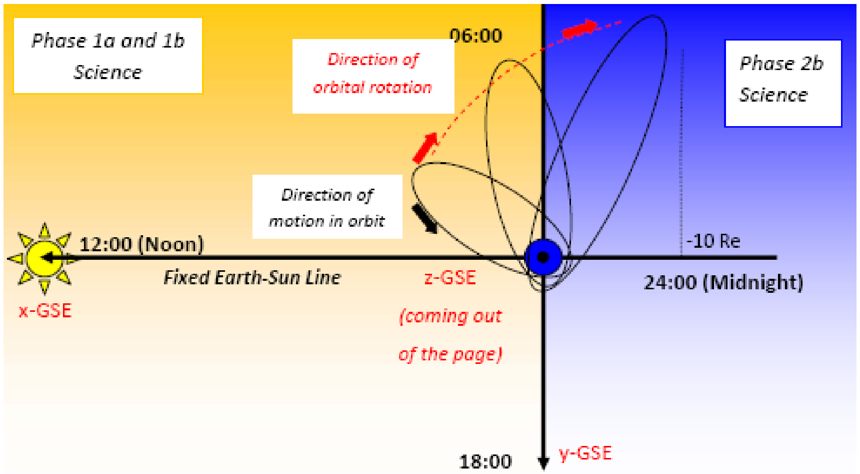

Figure 2. Schematic of GSE Rotating Frame with MMS Apogee Raisimg Region

The number of maneuvers required to perform the sequence must be minimized to reduce the campaign duration and lessen the operations burden, taking into account the burn durations, and the finite burn arc gravity loss and other losses (directly linked to fuel consumption) of the maneuvers. Satisfying these requirements represents a balancing act requiring numerical simulations of any proposed apogee raising strategy. The fewer the maneuvers, the greater the ΔV magnitudes, burn durations and finite burn losses.

THE MMS APOGEE RAISING TECHNIQUE

It was shown in the previous section that it is not feasible for MMS to raise apogee by an amount equal to 13 Re via a single maneuver. Instead, it will have to be accomplished incrementally via multiple perigee maneuvers. The question then is how best to break up the apogee raising process into multiple burns while meeting the various goals, guidelines and constraints discussed in the previous section. For an increment of apogee radius Δr_a, it can be shown that a convenient estimate of the required delta-V is provided by:

$$\Delta V_p \approx \frac{\mu}{4\, a^2\, V_p}\, \Delta r_a \tag{1}$$

where V_p is the pre-burn perigee velocity, a is the pre-burn semi-major axis, r_a is the apogee radius, and μ is Earth's gravitational parameter. Equation 1 provides the valuable insight that for a Δr_a of given magnitude, the larger the pre-burn orbit, the smaller the ΔV, due primarily to the factor a^{-2}. For example, raising a 12 Re apogee orbit by 1 Re to 13 Re would cost 35.45 m/sec, whereas raising apogee from 24 to 25 Re would cost just 9.88 m/sec. The significance of Equation 1 for our apogee raising method will be seen later in the paper.

Division of the Apogee Raising into Mean Motion Decrements of Equal Magnitude

A fundamental characteristic of our apogee raising method is the division of the full scenario into parts that are based on the synodic mean motion between the initial 1.2 by 12 Re orbit, period 0.99 day, and the final 1.2 by 25 Re orbit, period 2.78 days. If we denote the synodic mean motion between the initial and final orbits as Δn, then $\Delta n = n_i - n_f$ where n_i is the mean motion of the initial 0.99 day orbit and n_f is the mean motion of the final 2.78 day orbit. Note that for our case $n_i > n_f$. The intention, then, is to break up the Δn into equal segments by some number, j, to create segments of magnitude $\Delta n / j$. This $\Delta n / j$ represents the decrement of mean motion that will be achieved with each of the j segments. The change in orbit period corresponding to each segment is simply

$$\Delta T = \frac{2\pi\ j}{\left(n_i - n_f\right)} = \frac{2\pi\ j}{\Delta n} \tag{2}$$

Advancing through the segments allows determination of a new mean motion and period for the target orbit each time as follows

$$n_{to} = n_{curr} - \left[\frac{\left(n_i - n_f\right)}{j}\right] \tag{3}$$

where n_{curr} is the mean motion of the current orbit and n_{to} is the mean motion of the target orbit, and

$$T_{to} = \frac{2\pi}{n_{to}} \tag{4}$$

where T_{to} is the period for the target orbit. Taken together, Equations 2 through 4 describe the fundamental approach to segmentation of the ascent, define what the goal is for each segment in terms of net period change ΔT, and provide the resulting orbit period for each segment.

One more step of crucial importance is to achieve the ΔT in Equation 2 by executing two separate perigee maneuvers, such that each maneuver achieves a period change of $\Delta T/2$. The maneuver segmentation concept presented here is encapsulated schematically in Figure 3. In the figure R_i refers to the initial 1.2 by 12

Re orbit, and R_f refers to the final 1.2 by 25 Re orbit. Figure 3 depicts a four segment plan requiring a total of eight maneuvers.

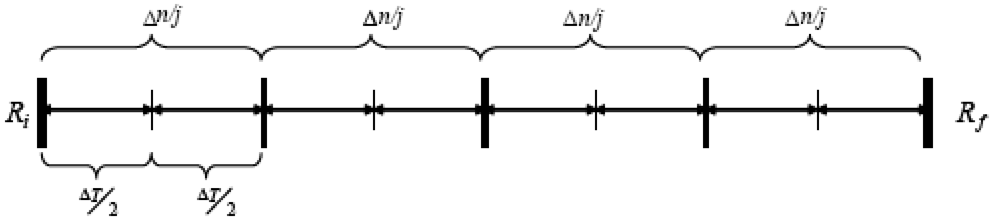

Figure 3. Schematic of the MMS Apogee Raising Segmentation Plan

Maneuver Order

A critical element of the apogee raising plan is the order in which the spacecraft will be maneuvered. The reasons for this are twofold: 1) the requirement that there be 24 hours between any two burns, and 2), the requirement that all spacecraft must always have at least one open, no-burn perigee following a maneuver before it maneuvers again. The former requirement provides maneuver calibration and re-planning time for operations, while the latter requirement provides the opportunity for orbit determination. Given that the period of the 1.2 by 12 Re orbit is very nearly 24 hours, the best way to ensure that these two requirements are met is to enforce a 1-2-3-4 order, then an open perigee passage with no spacecraft maneuvers, followed by the reverse order, 4-3-2-1, for each pair of maneuver legs. The numerals do not necessarily refer to the MMS spacecraft numbering scheme; rather, they should be thought of as a ‘maneuver slot’ assignment number. The slot #1 spacecraft will be the only spacecraft to maneuver for the given perigee passage. The slot #2 holder will be the sole spacecraft to maneuver on the following perigee passage, and so on. Initially, whichever of the spacecraft leads as the constellation approaches perigee (i.e., the spacecraft with the most advanced true anomaly) will occupy the #1 slot. For example, if MMS4 happened to be in the lead, it would get assigned slot #1 for the first maneuver leg, but then be the last to execute its second maneuver leg, some nine revolutions later. The ordering scheme is summarized in Table 1.

Table 1. Nominal Apogee Raising Maneuver Order Scheme: One Full Sequence

	Leg 1					Leg 2				
S/C Slot #	Rev 1	Rev 2	Rev 3	Rev 4	Rev 5 OD	Rev 6	Rev 7	Rev 8	Rev 9	Rev 10 OD
1	ΔT/2	–	–	–	–	–	–	–	ΔT/2	–
2	–	ΔT/2	–	–	–	–	–	ΔT/2	–	–
3	–	–	ΔT/2	–	–	–	ΔT/2	–	–	–
4	–	–	–	ΔT/2	–	ΔT/2	–	–	–	–

Note: ΔT/2 indicates the period change goal of the perigee maneuver. “–” indicates no maneuver.

Table 1 assumes that at the beginning of Revolution #1 (Rev #1), all four spacecraft are in phase at perigee and at the same epoch. The maneuvers occur at perigee, i.e., at the beginning of the Rev shown. All spacecraft have at least one no-burn perigee between each of its maneuvers. Because the orbit periods involved are never less than approximately 24 hours, there is at least one day between all maneuvers. The sequence shown in Table 1 repeats until all apogee raising maneuvers are complete, when all specified mean motion decrements are accomplished and the apogee radius is 25 Re. Table 1 represents one full sequence of maneuvers that taken together complete one mean motion decrement segment for all four spacecraft. One full sequence is considered to have two legs (as indicated by the table); each leg completes the period change goal $\Delta T/2$. Finally, the use of the maneuver order described above, when combined with having each leg of a sequence achieve a period change of $\Delta T/2$, contributes one more dynamic effect important to this technique, which is spacecraft re-phasing.

Spacecraft Re-phasing

A most interesting behavior of the apogee raising method described above is that, while the spacecraft spread out initially, they come back together and are again in phase by the end of a two-leg maneuver sequence. This behavior repeats itself for each of the 2-leg maneuver sequences performed to achieve the full apogee raising. It is a natural outcome due to two of the method's features: 1) having both burns achieve an identical period change goal (i.e., $\Delta T/2$), and 2), having the spacecraft maneuver in reverse order during the second leg of a sequence. Though it is not mandated by any requirement, this "automatic" re-phasing effect proves convenient for the mission should there be some reason during flight—such as a disabling spacecraft contingency or change in science goals—that the apogee raising all the way to 25 Re not be completed.

This behavior is explained in the equations (with reference to Table 1). At the beginning of any 2-burn sequence, all four spacecraft have the same orbit period, T_1,with a mean motion $n_1 = 2\pi / T_1$. When the Slot #1 spacecraft executes a perigee maneuver at the beginning of Rev #1, its period becomes $T_2 = T_1 + \Delta T$ (substituting the symbol ΔT for the orbit period change $\Delta T / 2$ given in Table 1). As seen from Table 1, Slot #1 spacecraft's period remains the same through Rev #8. At the beginning of Rev #9, the Slot #1 spacecraft executes its second maneuver to achieve another period change equal to ΔT. Its period thus becomes $T_3 = T_2 + \Delta T = T_1 + 2\Delta T$ for Rev #9. The total amount of time for Slot #1 to complete 9 full revolutions is $8T_2 + T_3$. Computing these totals for the spacecraft of all Slots yields the following set of four equations

$$8T_2 + T_3 = 8(T_1 + \Delta T) + T_1 + 2\Delta T = 9T_1 + 10\Delta T \tag{5}$$

$$T_1 + 6T_2 + 2T_3 = T_1 + 6(T_1 + \Delta T) + 2(T_1 + 2\Delta T) = 9T_1 + 10\Delta T \tag{6}$$

$$2T_1 + 4T_2 + 3T_3 = 2T_1 + 4(T_1 + \Delta T) + 3(T_1 + 2\Delta T) = 9T_1 + 10\Delta T \tag{7}$$

$$3T_1 + 2T_2 + 4T_3 = 3T_1 + 2(T_1 + \Delta T) + 4(T_1 + 2\Delta T) = 9T_1 + 10\Delta T \tag{8}$$

All four spacecraft complete 9 full revolutions and two maneuver legs in an identical time equal to $9T_1 + 10\Delta T$.

It also happens that the spacecraft are completing 18π radians in that period of $9T_1 + 10\Delta T$. The change in mean anomaly, M, over a full orbit period T is

$$M = \int_T n \cdot dt \tag{9}$$

The integral in Eq. 9 equals to 2π radians. Similarly a set of equations that integrate the total change in the mean anomaly of each spacecraft through the maneuver sequence from Table 1, nondimensionalizing time with respect to the various orbital periods through nine revolutions are

$$M_{SC1} = \int_{t_0+T_1}^{t_0+T_1+8(T_2)} n_2 dt + \int_{t_0+T_1+8(T_2)}^{t_0+T_1+8(T_2)+T_3} n_3 dt \tag{10}$$

$$M_{SC2} = \int_{t_0}^{t_0+(T_1)} n_1 dt + \int_{t_0+(T_1)}^{t_0+(T_1)+6(T_2)} n_2 dt + \int_{t_0+(T_1)+6(T_2)}^{t_0+(T_1)+6(T_2)+2(T_3)} n_3 dt \tag{11}$$

$$M_{SC3} = \int_{t_0}^{t_0+2(T_1)} n_1 dt + \int_{t_0+2(T_1)}^{t_0+2(T_1)+4(T_2)} n_2 dt + \int_{t_0+2(T_1)+4(T_2)}^{t_0+2(T_1)+4(T_2)+3(T_3)} n_3 dt \tag{12}$$

$$M_{SC4} = \int_{t_0}^{t_0+3(T_1)} n_1 dt + \int_{t_0+3(T_1)}^{t_0+3(T_1)+2T_2} n_2 dt + \int_{t_0+3(T_1)+2T_2}^{t_0+3(T_1)+2T_2+4(T_3)} n_3 dt \tag{13}$$

where t_0 is the initial epoch of the sequence, the limits indicate the time range where the following mean motions

$$n_1 = \frac{2\pi}{T_1}, \quad n_2 = \frac{2\pi}{T_1 + \Delta T}, \quad n_3 = \frac{2\pi}{T_1 + 2\Delta T} \tag{14}$$

apply, and the orbit periods are defined as

$$T_2 = T_1 + \Delta T \qquad T_3 = T_1 + 2\Delta T . \tag{15}$$

The mean motions for the initial, medial, and final orbits of a sequence (Eq. 14) remain constant between maneuvers. Evaluating the integrals of Equations 10 through 14 over the nine revolutions yield the following results:

$$M_{SC1} = M_{SC2} = M_{SC3} = M_{SC4} = 18\pi \tag{16}$$

Over a complete sequence assuming no perturbations, the spacecraft have all experienced an equal total change in mean anomaly in precisely the same amount of time. In summary, the spacecraft spread out in mean anomaly over the course of the first leg, but by reversing the order of maneuvers for the second leg, and performing period changes of the same magnitude as those achieved in the first leg, the separations are reduced to zero by the time the last spacecraft completes its second leg maneuver. Since by that point all spacecraft again have the same orbit periods, they remain in phase. The entire process repeats for as many sequences as specified to complete the segments of synodic mean motion decrements.

Apogee Raising Simulation Results

The apogee raising method described above—in combination with MMS finite burn and thruster models—was simulated in detail employing extensive scripting language code developed within the commercial-off-the-shelf (COTS) mission design tool called Freeflyer[4].[*] The simulation results suggest that a total of eight maneuvers (four synodic mean motion decrement sequences) is reasonable for the MMS mission. Our baseline test case had a start epoch of September 2, 2016 with the apogee raising complete 62 days later. Burn time for the 8 maneuvers ranged from 21.7 min to 36.0 min. Cumulative ΔV for the eight finite burns amounted to 254.5 m/sec, reflecting an average finite burn loss of about seven percent. Simulations showed that all four spacecraft finish the ascent very nearly in the same plane; differences in inclination, ascending node, and argument of perigee are no greater than approximately ±0.02 deg. Simulations also demonstrated not only the efficacy but also the necessity of the 1-2-3-4-4-3-2-1 maneuver order. Repeating a 1-2-3-4 order, for instance, shows that all spacecraft acquire a 25 Re orbit as intended, but they do not regroup; rather they remain spread out. An inertial frame orbit plot showing a representative MMS eight burn scenario appears in Figure 4.

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[*] http://www.ai-solutions.com/freeflyer/

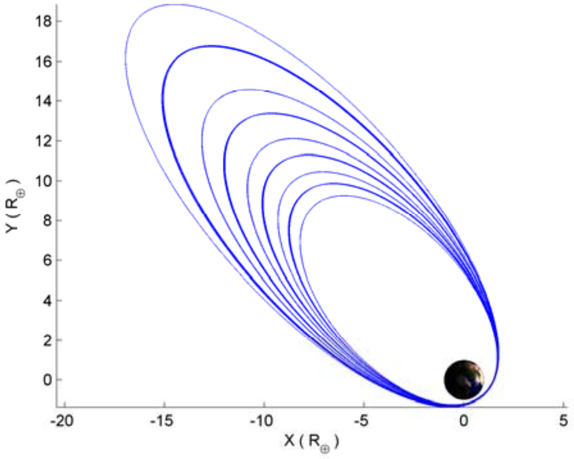

Figure 4. Inertial Frame Plot of MMS 8-burn Apogee Raising Scenario

In Figure 4, the orbits are prograde (counter-clockwise motion) and shown to scale; the heavier blue orbit traces correspond to the orbits with more revolutions than those with the lighter traces. For this case, the elapsed time between the first of the quartet's 32 maneuvers and the last is approximately 62 days. MMS orbits depicted in the rotating GSE frame appear later in the paper.

Table 2 provides some key parameters from a representative apogee raising case employing the technique described. Shown are the apogee radii and orbit period following each burn, and the ΔVs, burn times, and fuel required to achieve them. The eight impulsive ΔVs sum to 238.5 m/sec, whereas the finite burn ΔVs sum to 254.8 m/sec. Finite burn times vary from 21.7 to 36.0 min; total fuel usage sums to 116.9 kg.

Table 2. Representative MMS 8-burn Apogee Raising Case Key Parameters

Burn #	Apogee Radius (Re)	Period (days)	ΔV^{*} (m/s)	$\Delta V^{\dagger}$ (m/s)	Burn Time (min)	Fuel (kg)
1	12.83	1.09	28.7	28.8	23.7	13.9
2	13.63	1.19	24.7	26.2	21.7	12.5
3	14.78	1.33	30.9	33.4	27.5	15.7
4	15.89	1.47	25.8	27.1	22.4	12.6
5	17.61	1.69	34.0	37.1	30.6	16.9
6	19.26	1.92	27.0	28.6	23.6	12.9
7	22.22	2.35	38.9	43.5	36.0	19.3
8	25.00	2.78	28.5	30.1	24.9	13.1

* Impulsive †Finite Burn

Figure 5 below provides a plot showing inter-spacecraft ranges—or separations—in kilometers, from a software simulation of the apogee raising technique. Color-coded separation data for all six possible MMS pairs are plotted.

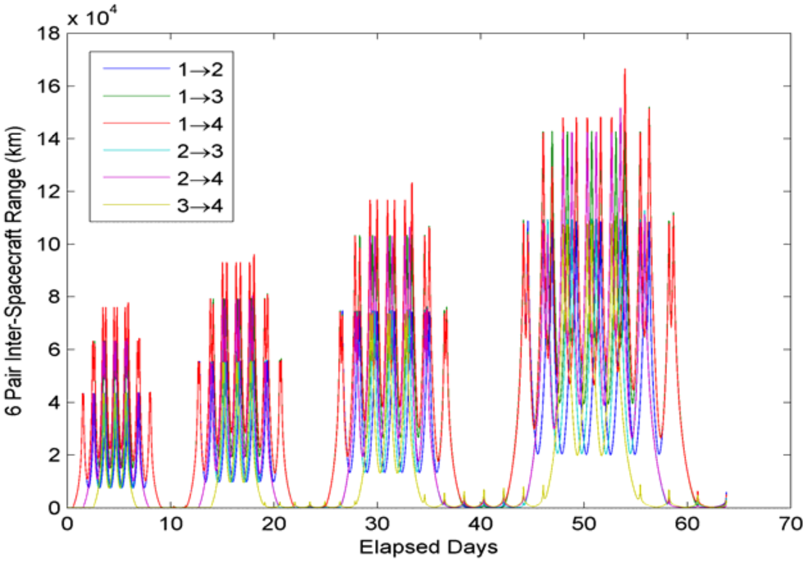

Figure 5. Spacecraft Separations for All MMS Pairs Show Re-phasing Every Other Maneuver

It can be seen from Figure 5 that the separations drop to virtually zero levels following every maneuver sequence. At these points the spacecraft are both re-grouped and in phase, and would remain so barring further maneuvers. However, as the spacecraft each perform a burn following such re-groupings they separate again. At the end of four such sequences, the apogee raising is complete. Notice also that the growth in separations from one maneuver sequence to the next is due to the fact that the orbits themselves are growing in size and the possible separation distances follow suit.

Figure 6 shows the growth in orbit period and apogee radius over eight maneuvers in this normalized data chart. Notice that for constant mean motion decrements and a relatively flat ΔV profile, each 2-burn sequence provides progressively larger increases in apogee radius. This outcome is predicted by Equation 1. Notice also that orbital period increases even more steeply with each 2-burn sequence—a most positive effect of the technique. A contrary apogee raising technique emphasizing large period increases early in the campaign would suffer from significantly larger, longer burns and the attendant finite burn losses.

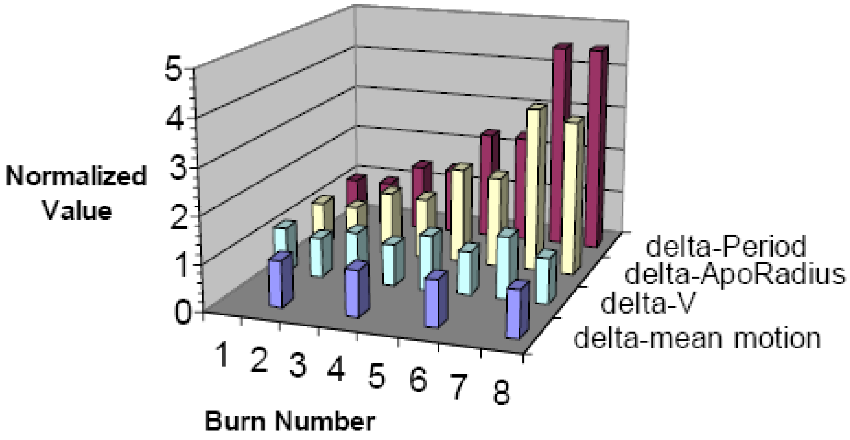

Figure 6. Orbit Period and Apogee Radius Growth over Eight Burns

Figure 7 below shows the apogee raising orbit plots for each of the four spacecraft. The views are to scale looking down on the GSE-frame X-Y plane from the North Ecliptic Pole; hence they represent projections of the orbits onto the ecliptic plane.

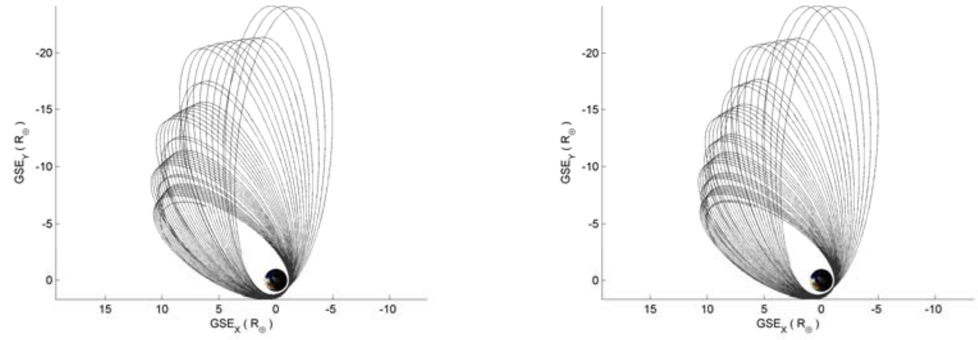

Fig. 7a. MMS1

Fig. 7b. MMS2

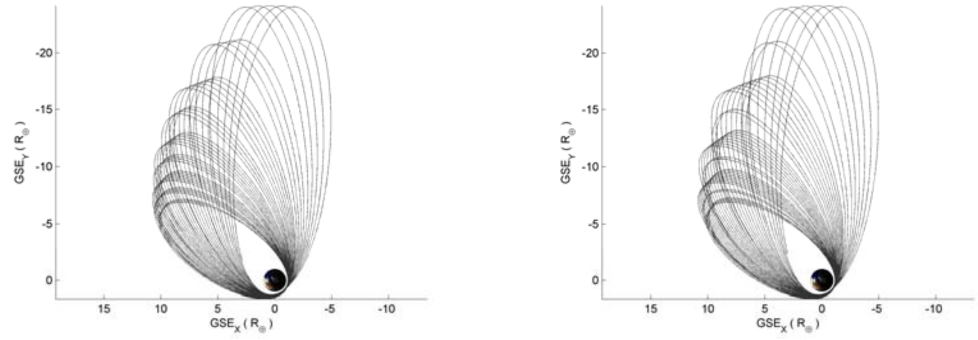

Fig. 7c. MMS3

Fig. 7d. MMS4

Figure 7. MMS4 8-burn Apogee Raising Orbits: GSE-frame X-Y Plane Projection

Note that spacecraft orbital motion is counterclockwise, while the line of apsides rotates clockwise within the GSE frame. Close examination of the plots reveals that they follow the pattern that would be expected by the maneuver sequence detailed in Table 1. The apogee raising case depicted here requires 62 days to complete. Notice also that for all four spacecraft the campaign concludes well before the apogee vector GSE-frame X-component reaches –10 Re (approximately 0 Re or better was achieved by all four spacecraft), thus satisfying the deadline requirement.

Post Apogee Raising Formation Re-initialization

The apogee raising algorithm permits re-phasing the spacecraft as arbitrarily close as desired, i.e., within one kilometer. However, the goal is to re-phase the spacecraft in a so-called ‘string of pearls’ configuration where they are all in basically the same orbit but deliberately spread out somewhat in true anomaly, such that in the apogee region they are separated from one another by distances on the order of 10^2 to 10^3 kilometers. From this configuration, the tetrahedral formation can be re-initialized via a Lambert targeter two-burn algorithm, with each burn located on either side of apogee. Both the re-phasing to a string of pearls configuration and subsequent formation re-initialization have been simulated successfully. In the preliminary testing, the Lambert method re-formation ΔVs computed are modest enough to fit within the MMS propellant budget. However a more thorough analysis and optimization of the formation re-initialization problem remains for later study.

OFF-NOMINAL MANEUVERS: SHORT TERM RECOVERY METHODS

Following the development of the MMS apogee raising method, an initial study of off-nominal maneuver recovery and contingency scenario recovery techniques was completed. Off-nominal maneuvers or maneuver anomalies qualifying as contingencies, can be described as the spacecraft having achieved an orbit period change of $k\Delta T$[*], where k lies within $0 \leq k < 1$ for underburns, and $k > 1$ for overburns. (Recall that ΔT represents the period change goal for the desired maneuver.) The special case of $k = 0$ indicates that, for whatever reason, the spacecraft did not perform a burn as scheduled. The case of $k < 0$, i.e., an accidental retrograde burn, was not considered for this study.

Recoveries for an entire apogee raise scenario fall into two main categories—short term and long term. For any i^{th} two-burn sequence (such as outlined in Table 1), a recovery is considered short term if full recovery can be achieved no later than the end of the $i^{th} + 1$ sequence. Generally, this requires that the spacecraft be ready to maneuver again following the orbit determination (OD)-dedicated revolution after its off-nominal or anomalous burn. The definition of short-term recovery requires meeting one of the two following criteria:

1. Off-nominal or anomalous burns incurred during the first leg of the i^{th} sequence are fully recoverable by the end of the second leg.
2. Off-nominal or anomalous burns incurred during the second leg of the i^{th} sequence are fully recoverable by the end of the $i^{th} + 1$ sequence.

Long term recoveries are defined as any i^{th} sequence anomaly that cannot be completely recovered prior to the end of the $i^{th} + 1$ sequence. Typically, such a case indicates a problem requiring significant time to resolve. Since the further assumption is that the other spacecraft would not mark time but continue with their maneuvers as scheduled, the anomalous spacecraft would then fall behind the apogee raising pace of the others and need to play catch-up in some manner; this spacecraft is referred to as a 'Straggler.'

Guidelines applied to the development of recovery techniques are as follows:

- Where possible avoid simultaneous maneuvers, i.e., continue to observe the maneuver spacing requirements set forth for the apogee raising technique
- Avoid retrograde maneuvers
- Preserve the requirement for OD-dedicated revolutions following maneuvers
- Spacecraft experiencing short term recoveries should have their periods and relative phasing re-constituted by the end of the appropriate sequence; for long term recoveries, the affected spacecraft should complete the ascent to a 25 Re orbit and be re-phased with the others as soon as feasible

Short Term Recovery: Two Burn Method

A short term contingency recovery scenario is where an off-nominal burn $k\Delta T$ occurs within the first leg of a two-burn sequence, and a recovery is to be achieved by the end of the second leg. To develop the scenario, assume the off-nominal burn has occurred for the Slot #1 spacecraft. It is proposed that the recovery involve two burns. One is a correction burn called $a\Delta T$, inserted at the beginning of Rev #5 (this is

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[*] We continue the substitution of the symbol ΔT for a single leg period change, in lieu of ΔT / 2.

nominally the OD-only revolution included for the benefit of the Slot #4 spacecraft (see Table 1)).[*] The second burn occurs on Rev #9, where the nominal second burn for Slot #1 spacecraft is located. This means that an adjustment needs to be made to what would have been, nominally, a burn ΔT; but since its magnitude needs to be different, call it $b\Delta T$ (see Table 3).

Table 3. Slot #1 Spacecraft 2-burn Short Term Recovery from Initial Off-nominal Burn

Slot #	Rev 1	Rev 2	Rev3	Rev 4	OD	Rev 6	Rev 7	Rev 8	Rev 9	OD
1	$k\Delta T$	–	–	–	$a\Delta T$	–	–	–	$b\Delta T$	–

To solve for the coefficients a and b, observe that there are two constraint equations that apply. First, satisfying the constraint that all spacecraft have the same final period means that the following obtains:

$$k\Delta T + a\Delta T + b\Delta T = 2\Delta T \tag{17}$$

Given that following the burn $k\Delta T$, the value of k is a known, fixed quantity, Equation 17 simplifies to a linear equation with unknowns a and b:

$$a + b = 2 - k \tag{18}$$

Next, to meet the re-phasing constraint, the following linear equation must also be satisfied:

$$9k\Delta T + 5a\Delta T + b\Delta T = 10\Delta T \tag{19}$$

Solving the pair of Equations 18 and 19 via substitution yields the solutions:

$$a = 2(1 - k) \quad \text{and} \quad b = k \tag{20}$$

With k known, the coefficients a and b are easily computed.

The same basic approach can be taken for treating first leg anomalies for spacecraft Slot #2 through #4, once deciding on placement for the correction burns. Following the example given for Slot #1, a table of off-nominal first leg burns and their recovery burns for all four spacecraft is constructed as shown in Table 4. Notice that in all cases the $a\Delta T$ burn is placed on Rev #5 and the $b\Delta T$ burn occurs on the same revolution as the nominal second leg burn (compare Table 1).

Table 4. Two-burn Recovery Scenarios for First Leg Off-Nominal Burn

	Leg 1					Leg 2				
S/C Slot #	Rev 1	Rev 2	Rev 3	Rev 4	Rev 5 OD	Rev 6	Rev 7	Rev 8	Rev 9	Rev 10 OD
1	$k\Delta T$	–	–	–	$a\Delta T$	–	–	–	$b\Delta T$	–
2	–	$k\Delta T$	–	–	$a\Delta T$	–	–	$b\Delta T$	–	–
3	–	–	$k\Delta T$	–	$a\Delta T$	–	$b\Delta T$	–	–	–
4	–	–	–	$k\Delta T$	$a\Delta T$	$b\Delta T$	–	–	–	–

Note: ΔT indicates the period change goal of the perigee maneuver. "–" indicates no maneuver.

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[*] The basic idea is that a correction burn needs to be inserted at a point early enough within the leg to make up for the re-phasing losses due to the off-nominal initial burn. These losses simply cannot be made up by a burn placed at the very end of the second leg if a fully re-phased recovery is to be achieved by the end of that second leg.

For all slots of Table 4, the linear constraint Equation 18 applies. From considering Table 4, linear equations satisfying the re-phasing constraint are unique for each slot, and can be written, after eliminating the constant ΔT throughout, as the following set given in order from Slot #1 through Slot #4:

$$9k + 5a + b = 10 \tag{21}$$

$$8k + 5a + 2b = 10 \tag{22}$$

$$7k + 5a + 3b = 10 \tag{23}$$

$$6k + 5a + 4b = 10 \tag{24}$$

Observe that the solutions $a = 2(1 - k)$ and $b = k$ (Equation 20) satisfy each of the Equations 21 through 24. One thing to notice for Slot #4 in Table 4 is that there are no burn-free revs after Rev #4 and Rev #5. The study showed, however, that in the event of a Slot #4 off-nominal burn, a no-burn OD rev could be inserted after both Rev #4 and Rev #5 and not affect the form of the solutions (Equation 20). Nor would these insertions affect the quartet's re-phasing results. Taking such action would however increase the number of revs by two for all spacecraft for the affected sequence, thereby also increasing the duration of the apogee raising campaign by an amount $2T + 2k\Delta T + a\Delta T$, where T is the period of the orbit at the very beginning of that sequence.

Second leg anomalies can be treated in a manner similar to the foregoing. The chosen locations for the recovery burns to correct an anomaly occurring in the second leg of a sequence *i* appear in Table 5.

Table 5. Two-burn Recovery Scenarios for Second Leg Off-nominal Burn

	Leg 2, Sequence *i*					Leg 1, Sequence *i* + 1				
S/C Slot #	Rev 6	Rev 7	Rev 8	Rev 9	Rev 10 OD	Rev 1	Rev 2	Rev 3	Rev 4	Rev 5 OD
1	–	–	–	$k\Delta T$	$\alpha\Delta T$	$(1+\beta)\Delta T$	–	–	–	–
2	–	–	$k\Delta T$	–	$\alpha\Delta T$	–	$(1+\beta)\Delta T$	–	–	–
3	–	$k\Delta T$	–	–	$\alpha\Delta T$	–	–	$(1+\beta)\Delta T$	–	–
4	$k\Delta T$	–	–	–	$\alpha\Delta T$	–	–	–	$(1+\beta)\Delta T$	–

Note: ΔT indicates the period change goal of the perigee maneuver. "–" indicates no maneuver.

In this case, the linear equations of constraint are:

$$k\,\Delta T + \alpha\Delta T + \beta\Delta T = \Delta T \tag{25}$$

and for the Slots #1 through #4 in order:

$$11\Delta T + 3k\Delta T + 2\alpha\Delta T + \beta\Delta T = 14\Delta T \tag{26}$$

$$11\Delta T + 5k\Delta T + 3\alpha\Delta T + \beta\Delta T = 16\Delta T \tag{27}$$

$$11\Delta T + 7k\Delta T + 4\alpha\Delta T + \beta\Delta T = 18\Delta T \tag{28}$$

$$11\Delta T + 9k\Delta T + 5\alpha\Delta T + \beta\Delta T = 20\Delta T \tag{29}$$

Solving the above equations through substitution yields the solutions applicable to all four slots:

$$\alpha = 2\,(1 - k) \quad \text{and} \quad \beta = k - 1 \tag{30}$$

As before, once k is known from the off-nominal burn results, the coefficients α and β are simple to compute and hence the magnitudes of the correction burns are specified. For this scheme the maneuver $\beta\Delta T$ is applied at the nominal locations of the first leg burns for sequence $i + 1$. The idea is to combine the two maneuvers such that a single maneuver $(1+\beta)\Delta T$ is performed. However, note from Equation 30 that $(1 + \beta) = k$. So, simply applying a maneuver equal in magnitude to that of the off-nominal burn of the ith sequence has the effect of simultaneously catching up the goals of the ith sequence and completing the first leg goal of the ith + 1 sequence. This result is verified by considering the net period increase of the three burns $k\Delta T$, $\alpha\Delta T$, and $(1+\beta)\Delta T$ as follows:

$$k\Delta T + \alpha\Delta T + (1 + \beta)\Delta T = k\Delta T + 2(1 - \mathrm{k})\Delta T + (1 + \mathrm{k} - 1)\Delta T = 2\Delta T \tag{31}$$

Lastly, if the off-nominal burn is an underburn, that is $k < 1$, then β is negative. Given that, a β-only burn would be retrograde if it were not combined with the first ΔT burn of the ith + 1 sequence. The advantage here is that the "blended" correction burn $(1 + \beta)\Delta T$ yields a net posigrade result. However, if k is an overburn ($k > 1$), then the $\alpha\Delta T$ is unavoidably a retrograde burn.

Short Term Recovery: Three Burn Method

The two burn recovery method solutions can in some cases, depending on the value of k, produce maneuvers that are either much larger than a nominal ΔT maneuver and/or maneuvers that are retrograde. Either situation is not desirable. For that reason, an approach involving three recovery burns, which will be referred to by the variables x, y, and z, was developed through exploring reasonable placements for the burns within a sequence. The system developed for Slots #1 through #4 is summarized in Table 6.

Table 6. Three-burn Recovery Scenarios for First Leg Off-nominal Burn

	Leg 1					Leg 2				
S/C Slot #	Rev 1	Rev 2	Rev 3	Rev 4	Rev 5 OD	Rev 6	Rev 7	Rev 8	Rev 9	Rev 10 OD
1	$k\Delta T$	–	$x\Delta T$	–	$y\Delta T$	–	–	–	$z\Delta T$	–
2	–	$k\Delta T$	–	$x\Delta T$	–	$y\Delta T$	–	$z\Delta T$	–	–
3	–	–	$k\Delta T$	–	$x\Delta T$	–	$y\Delta T$	–	$z\Delta T$	–
4	–	–	–	$k\Delta T$	–	$x\Delta T$	–	$y\Delta T$	–	$z\Delta T$

Note: ΔT indicates the period change goal of the perigee maneuver. "–" indicates no maneuver.

Note that for Slot #1 the $z\Delta T$ is located on Rev #9 rather than Rev #7. This is deliberate to show that there is latitude in placement of the burns for this method. The equations of constraint for this system are:

$$k + x + y + z = 2 \tag{32}$$

$$7x + 5y + z = 10 - 9k \tag{33}$$

$$6x + 4y + 2z = 10 - 8k \tag{34}$$

$$5x + 3y + z = 10 - 7k \tag{35}$$

$$5x + 3y + z = 12 - 7k \tag{36}$$

The maneuver operations analyst can choose the magnitude for one of the variables x, y, or z, yielding a set of two linear equations determining the magnitude of the remaining two variables. For example, in the event of anomalous burn the x recovery burn might be chosen to be of modest size as a precaution, as it would be the first to occur following the anomalous burn.

The three-variable approach for off-nominal burns occurring in the second leg of a sequence *i* is shown in Table 7. Correction burns mostly fall within the first leg of sequence *i* + 1. For Slot #4 in Table 7, $z\Delta T$ has on option been placed on Rev #4 rather than Rev #2.

Table 7. Two-burn Recovery Scenarios for Second Leg Off-nominal Burn

	Leg 2, Sequence *i*					Leg 1, Sequence *i* + 1				
S/C Slot #	Rev 6	Rev 7	Rev 8	Rev 9	Rev 10 OD	Rev 1	Rev 2	Rev 3	Rev 4	Rev 5 OD
1	–	–	–	$k\Delta T$	–	$x\Delta T$	–	$y\Delta T$	–	$z\Delta T$
2	–	–	$k\Delta T$	–	$x\Delta T$	–	$y\Delta T$	–	$z\Delta T$	–
3	–	$k\Delta T$	–	$x\Delta T$	–	$y\Delta T$	–	$z\Delta T$	–	–
4	$k\Delta T$	–	$x\Delta T$	–	$y\Delta T$	–	–	–	$z\Delta T$	–

Note: ΔT indicates the period change goal of the perigee maneuver. "–" indicates no maneuver.

The system of equations corresponding to Table 7, in order from Slot #1 to Slot #4, are:

$$k + x + y + z = 2 \tag{37}$$

$$5x + 3y + z = 12 - 7k \tag{38}$$

$$5x + 3y + z = 10 - 7k \tag{39}$$

$$6x + 4y + 2z = 10 - 8k \tag{40}$$

$$7x + 5y + z = 10 - 9k \tag{41}$$

An example of how the 3-burn approach can be used is shown by the plot in Figure 8.

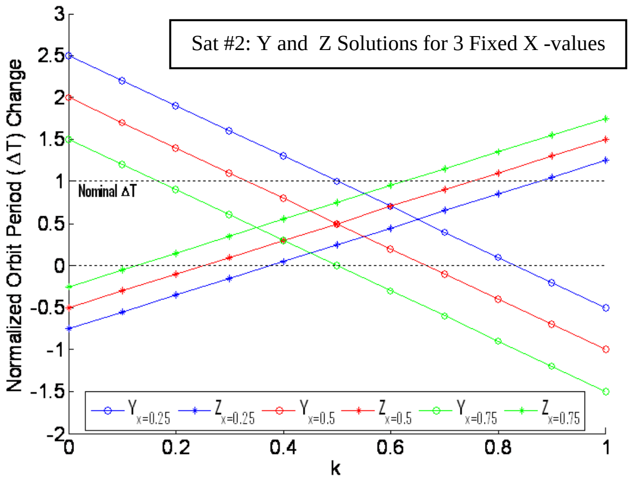

Figure 8. Three-Burn Method Cases for X- Burns Fixed at 0.25, 0.5, and 0.75

Figure 8 depicts the *y*-burn and *z*-burn solutions for three different user-chosen values for the *x*-burn, based on the solutions of Equations 32 and 34. The plot indicates that if *k* is large, it is likely more favorable to choose a small value for *x*. If, however, *k* is small, it appears choosing a larger value for the *x*-burn would be better.

Comparison of Two Burn and Three Burn Methods

The two burn and three burn methods represent alternative approaches the maneuver analyst can bring to bear on the problem of recovering from off-nominal or anomalous burns. In any given situation, the analyst would want to develop the solutions for both methods and compare the results to arrive at the most favorable solution to the recovery problem. An example is shown in Figure 9.

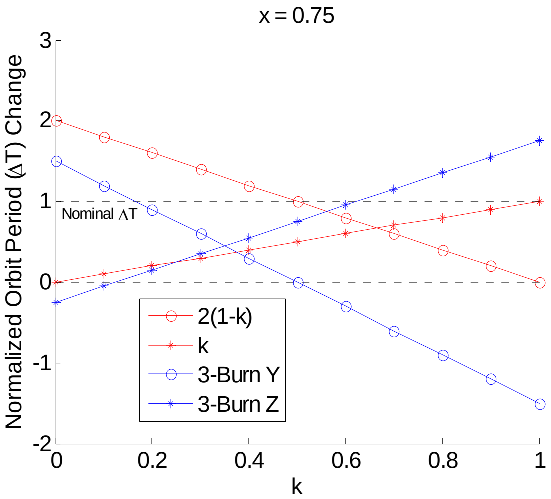

Figure 9. Sat #2: Two Burn Solution vs. Three Burn Solution with X = 0.75

Examining Figure 9, the analyst would note that the 3-burn method with the selection of $x = 0.75$ yields acceptable solutions for *y* and *z* (shown in blue) only over the domain where $0.1 \le k \le 0.5$. Outside of that span, the 3-burn method yields undesirable retrograde burns. The 2-burn solutions (shown in red) are attractive for $k \ge 0.5$, but are not for $k < 0.1$, due to the very large *a*-burn (i.e., $2(1 - k)$). For $k < 0.1$, one should consider a 3-burn method with a different choice of *x*. In fact, if $x = 1$ is chosen, then from Equations 32 and 34 the *y*-burn would range from 1 to 0.7, and the *z*-burn from 0 to 0.2, as *k* varies from 0 to 0.1.

OFF-NOMINAL MANEUVERS: LONG TERM RECOVERY METHOD

As previously mentioned, a long term method will be needed should one (or more) of the spacecraft suffer a serious contingency that pauses its apogee raising campaign until the contingency is resolved. The spacecraft having a contingency is referred to as a ‘Straggler’ because it would lag behind the others. A working assumption is that the unaffected spacecraft reach the destination 25 Re orbit before the Straggler.

The solution to how the Straggler eventually catches up and re-phases with the others is basically a rendezvous technique.[5] The rendezvous technique is described with a simple example, referring to a ‘Destination’ orbit (which is the 25 Re orbit) with period T_D, a ‘Phasing’ orbit with period T_P, and a ‘Staging’ orbit with period T_S. The periods of these three orbits have the relationship $T_S < T_P < T_D$. The value of T_D is 4,000 min. The Phasing orbit is computed so that it will re-phase the Straggler with the rest of the formation already in the Destination orbit. The Staging orbit is an orbit at some level of the ascent from which the Straggler can jump onto the Phasing orbit in one maneuver. The Staging orbit period T_S is selected with the aforementioned period relationship in mind.

The proposal is to incrementally raise up the Straggler, from wherever it is, to the Staging orbit with a period of, say, $T_S = 3{,}000$ min. Once in the Staging orbit, a reasonable Phasing orbit—with period between 3,000 and 4,000 min—must be computed. The mean motion of the Destination orbit is $n_D = 360 \text{ deg} / T_D =$

0.09 deg/min, and assume the Straggler is at perigee with mean anomaly MA = 0 deg. It must also be known where the Formation spacecraft are within their 25 Re orbit at this same epoch; so, for the example assume they are at apogee with MA = 180 deg. Thus, the difference in Mean Anomaly between the Formation and the Straggler is ΔMA = 180 deg.

Since the Straggler must be at some future perigee to jump onto the Destination orbit in phase with the Formation, future times when the latter spacecraft will be at perigee must be known. For instance, they will be at perigee 3.5 revolutions hence, and so an analyst can compute how long it will take the formation to arrive there via

$$[\ 3\ (360\ \text{deg}) + 180\ \text{deg}\ ]\ /\ n_D\ = [\ 3\ (360\ \text{deg}) +\ \Delta\text{MA}\ ]\ /\ 0.09\ =\ 14{,}000\ \text{min} \tag{42}$$

The Straggler can catch the formation at that perigee passage in four revolutions if it is in a Phasing orbit with period

$$\text{T}_\text{P} = 14{,}000\ \text{min}\ /\ 4\ =\ 3{,}500\ \text{min} \tag{43}$$

The Straggler should do an apogee raise maneuver boosting its period by 500 min. That way it will return to perigee in 4 revs in the same time (14,000 min) that it takes the others to reach perigee, at the same epoch as the Straggler, in 3.5 revs of their larger orbit. Computing the mean motion of the Phasing orbit, and then its synodic mean motion with the Destination orbit, will show that the catch-up rate for the Straggler is 45 deg of mean anomaly per revolution. In fact the Phasing orbit is in an 8:7 resonance with the Destination orbit. Other solutions exist. For example, the Straggler could catch the formation in 3 revs if it were in a Phasing orbit with T_P = 3,333.33 min, which represents a 6:5 resonance and a catch-up rate of 60 deg of mean anomaly per rev. However when comparing solutions for MMS, it is important to be mindful of the trade-offs between catch-up time and the magnitudes of the burns needed to achieve both the Phasing orbit and the Destination orbit. Larger burns may be unattractive due to long burn times and larger finite burn losses.

In general, different solutions for the Phasing orbit can be computed from the heuristic expression

$$\text{T}_\text{P}\ =\ (1/R_P)\ [\ R_D\ (360\ \text{deg}) \pm \Delta\text{MA}\ ]\ /\ n_D \tag{44}$$

where R_D is the number of revolutions selected for the Formation, and R_P is the number of revolutions desired for the Straggler in the Phasing orbit. The ΔMA can be added or subtracted depending on whether the Formation is considered to be leading or following the Straggler. Solutions for T_P are only valid if they meet the criterion $\text{T}_\text{S} < \text{T}_\text{P} < \text{T}_\text{D}$.[*] In general, the orbit solutions found will not be in exact resonance, though often they may be near-resonant. The example above is of course fairly trivial. Calculations and Freeflyer simulations have both verified a variety of more realistic scenarios where $\Delta\text{MA} \neq 180$ deg when the Straggler is at perigee of its Staging orbit.

CONCLUSION

Two important aspects of MMS mission design were presented. The first dealt with the problem of raising the apogee of four spacecraft in a 1.2 by 12 Re highly elliptical orbit to 25 Re for the second science phase of the mission. The four spacecraft are to be independently raised such that they safely achieve the enlarged orbit in a safely separated string of pearls prior to re-initializing the tetrahedral formation. A successful technique for accomplishing this task was presented in detail. The technique involves two main parts. First, the synodic mean motion between the 1.2 by 12 Re orbit and the 1.2 by 25 Re orbit is divided into four equal segments. Second, the segments represent the orbit change goals that will be achieved via a

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[*] If $\text{T}_\text{P} < \text{T}_\text{S}$ a retrograde maneuver is implied (undesirable), followed by a large burn to achieve orbit T_D. If $\text{T}_\text{P} > \text{T}_\text{D}$, a possible large burn from orbit T_S is implied, as well as an undesirable follow-on retrograde burn back to orbit T_D.

two-burn sequence. Each burn of the sequence in turn achieves half of the overall orbit period change corresponding to the segment's desired change in mean motion. Additionally, the spacecraft perform the second burn in reverse order. The net effect is that though the spacecraft are spread out in mean anomaly following the first burn, the separations are entirely undone following completion of the second; this means the spacecraft are re-grouped and re-phased following each sequence repeatedly, leading to the desired end result. Reversing the order is also critical to meeting the requirement of enforcing 24-hour intervals between maneuvers. The effectiveness of the apogee raising technique was verified via software simulations that included detailed finite burn modeling.

The second area dealt with methods for addressing the impacts of off-nominal burns or other contingencies on the apogee raising campaign. A key feature of the apogee raising technique is that, though the four spacecraft spread out during the first part of a two-burn sequence, they become re-grouped and re-phased after completing the second burn. This behavior is repeated with every pair of burns through-out the ascent until the 25 Re orbit is reached. However, off-nominal or anomalous burns can perturb this behavior and prevent re-phasing; hence recovery techniques are required. Techniques for short term recoveries and long term recoveries were presented in detail. Recoveries that occur within the time-frames of a given single sequence or within the first half of the subsequent sequence are considered short term recoveries. Two-burn and three burn recovery methods were described for handling such cases. Long term recoveries refer to situations where a serious anomaly for a spacecraft result in its being left behind until its contingency is resolved. The affected spacecraft would then pursue an independent apogee raising campaign leading up to a specific phasing orbit for re-phasing with the others prior to its final apogee raising burn. The approach is essentially a type of rendezvous problem. It is also important to point out that the rendezvous recovery method is fundamentally the recovery technique of last resort for all contingency situations, serving as a back-up to the short term or other techniques if they become impractical or break down for any reason. The validity of the short term and long term techniques was also verified via software simulations.

Finally, future work will tackle issues such as the effects of multiple non-nominal burns and avoiding mutual close approaches during the apogee raising campaign. It is expected that, at a minimum, refinements to the techniques described here will be necessary, and quite possibly variant or derivative methods that may prove more suitable will be developed as well.

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