

Iterative Magnetometer Calibration

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This paper presents an iterative method for three-axis magnetometer (TAM) calibration that makes use of three existing utilities recently incorporated into the attitude ground support system used at NASA's Goddard Space Flight Center. The method combines attitude-independent and attitude-dependent calibration algorithms with a new spinning spacecraft Kalman filter to solve for biases, scale factor errors, nonorthogonal corrections to the alignment, and the orthogonal sensor misalignment. The method is particularly well-suited to spin-stabilized spacecraft, but may also be useful for three-axis stabilized missions given sufficient data to provide observability. Results are presented for tests using both simulated data and actual flight data from the ST5 and Aura missions.

I. Introduction

Recent work on the attitude ground support system^{1,2} used for mission support at the NASA Goddard Space Flight Center (GSFC) has included incorporation of an improved attitude-independent calibration utility for three-axis magnetometers (TAMs) as devised by Alonso and Shuster³ and, separately, a new Kalman filter for spin-stabilized spacecraft.^{4,5} A new attitude-dependent alignment estimation utility was added a few years ago.⁶ The current work describes how these three utilities have been combined to create a tool for TAM calibration. The method is motivated in part by earlier iterative TAM calibration experiments using data from the Upper Atmosphere Research Satellite (UARS) and Extreme Ultraviolet Explorer (EUVE) missions.⁷

The new method is particularly well-suited to spin-stabilized spacecraft where such a tool had been lacking. For three-axis stabilized spacecraft with more accurate sensors (e.g., star trackers and gyros), other TAM calibration algorithms are available; however, the method described here is still useful for preliminary calibration prior to calibration of the fine sensors. Preliminary calibration is valuable for missions such as Aqua, Aura, and the National Polar-orbiting Operational Environmental Satellite System (NPOESS) that require moderate accuracy from the coarse sensors to acquire and identify stars before transitioning to fine-pointing mode.

The purpose of TAM calibration is to improve estimates of the parameters needed to convert raw magnetometer measurements from sensor frame to body-frame observations of the instantaneous geomagnetic field. For the usual TAM sensor model, these parameters are the sensor bias and scale factor for each axis and the alignments of the axes. A coarse calibration often estimates only the biases. With sufficient data, an attitude-independent, *TAM-only* method can determine the biases, scale factors, and the nonorthogonal part of the alignment (a measure of the skewness of the three axes). Alonso and Shuster give an excellent survey of attitude-independent methods.⁸ When data from other sensors is available, the orthogonal misalignment can be determined relative to those other sensors.

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An additional set of parameters often is used to correct for contamination of the TAM measurements by stray fields from magnetic torquer coils. This TAM-torquer coupling is not considered in this paper; however, in passing we note that various missions have dealt with this problem in a variety of ways. One can statistically estimate the coupling matrix using an accurate star tracker and gyro-based attitude (e.g., RXTE and EUVE); one can measure the coupling matrix directly by methodically turning each torquer on and off (e.g., SWAS and WIRE); or one can avoid its use entirely if the torquers are guaranteed off for some reserved fraction of each data frame (e.g., Aqua and Aura).

Section II presents the steps of the iterative method and gives a brief overview of each of the three utilities used. Section III describes tests performed using both simulated data and actual flight data from the Space Technology 5 (ST5) and Aura missions. Section IV provides some conclusions.

II. Method

The new TAM calibration algorithm combines attitude-independent and attitude-dependent calibration utilities with an attitude estimator. These are applied in the following steps:

- A. Use an attitude-independent TAM calibration method to estimate
 - bias
 - scale factor
 - nonorthogonal part of the alignment

Readjust the TAM data with these parameters.

- B. Solve for the attitude history using data from the readjusted TAM and at least one other sensor. (If multiple sensors are available, designate one other than the TAM as primary. This will be referred to below as the independent sensor.)
- C. With the attitude history from Step B, use an attitude-dependent method to estimate the orthogonal misalignments of the TAM and independent sensor. Combine these with the nominal alignments to form a single TAM alignment. Readjust the TAM data.

Repeat steps B and C until no further significant changes are found in the alignment.

A. Attitude-Independent TAM Calibration

TAM data alone can be used to perform a self-calibration based on the consistency of the magnitudes of the observed and reference magnetic fields. No attitude, rate, or other sensor knowledge is needed. Besides being part of the iterative calibration described here, this step also is useful by itself, particularly during the first several hours of a mission when other sensors may not yet be powered on and before any postlaunch relative misalignments have been determined.

Not all the TAM parameters can be solved for with an attitude-independent method. Parameters that affect the magnitude of the measured field are observable; those that only affect the measured direction are not. The unconstrained 9-degree-of-freedom alignment/scale factor matrix can be factored into orthogonal and symmetric matrices. The orthogonal matrix represents rigid rotation of the TAM axes relative to the body frame; the symmetric matrix represents a nonorthogonality correction. The diagonal part of the symmetric matrix gives the scale factors, and the off-diagonal part gives the skewness of the three axes. This skewness is the deviation from 90 degrees of the separations between the X-Y, Y-Z, and Z-X axes, defined symmetrically so

the orthogonal alignment is unaffected. Using magnitudes only, it is clear that the orthogonal misalignment of the TAM will not be observable since vector inner products are frame-independent; however, the TAM biases, scale factor corrections, and skewness can be estimated.

The specific method used is Alonso and Shuster's TWOSTEP.³ This is a rigorous method to maximize the likelihood function formed from effective measurements consisting of the error in the squared magnitude of the field. The log of the likelihood function is quartic in the calibration parameters. TWOSTEP avoids the problem of multiple extrema for a quartic function by applying a centering method to separate the function into two parts (this step is similar to other published methods⁸). One part is quadratic in the parameters and yields an unambiguous first estimate. This estimate is used in the second step as a starting point for a Gauss-Newton iteration to solve for the correction needed to maximize the entire likelihood function.

B. Attitude Estimation

After correcting the TAM for bias, scale factor, and nonorthogonality errors, the TAM and other available sensors are used to determine the attitude as a function of time. There currently are three options for attitude estimation available in the NASA/GSFC ground support systems. Two of these options, a batch least-squares differential corrector and an extended Kalman filter (EKF), are appropriate for three-axis stabilized spacecraft with gyro rate data available. The third option is an EKF designed for spin-stabilized spacecraft (referred to as SpinKF).^{4,5} (There also are less accurate "single-frame" methods that are not considered here.)

The estimators for three-axis stabilized spacecraft have been used successfully for many years at GSFC for a large number of missions. The SpinKF estimator is a new utility being used for mission support for the first time this year. Previously, support for spinners was provided by a separate batch least-squares utility that assumed a constant spin direction throughout the data batch; it did not account for torques or nutational motion. SpinKF is more versatile: it supports a wide variety of attitude and rate sensor input data, can account for environmental and control torques, and integrates the dynamics equations using a 4th-order Runge-Kutta method. (Although the previous batch utility did not include dynamics modeling, it did solve for a variety of sensor biases that are not included in SpinKF. The current work in part addresses that lack.)

The SpinKF estimator is unique in that it uses a novel state vector based on the angular momentum vector in the body frame, the angular momentum in an inertial frame, and a rotational phase angle.⁹ The angular momentum is slowly varying, assuming reasonably sized torques, so all the rapid variability is isolated in the phase angle state element. This property makes this representation superior to quaternion and rate variables or any other representation where more than one state element are rapidly varying. This advantage is significant in the filter's state propagation step where the numerical errors will be smaller when integrating the equations of motion.

C. Attitude-Dependent TAM Calibration

With the approximate attitude history from the previous step, misalignments are determined that minimize the TAM residuals. Of course, this attitude will have errors arising from the, as yet, incomplete calibration of the TAM; however, these errors will decrease on each iteration.

The method used to solve for the misalignment, given an attitude, is based on the QUEST quaternion estimation algorithm¹⁰ and is referred to as ALIQUEST.⁶ The QUEST algorithm is a well-known, efficient, and reliable method to solve the Wahba problem¹¹ to minimize the loss

function, L , as a function of the attitude matrix, A ,

$$L(A) = \sum_i \left[\mathbf{v}_i^{body} - A \mathbf{v}_i^{ref} \right]^2 \quad (1)$$

where vector \mathbf{v}_i^{body} is the observation unit vector for sensor i expressed in the body frame, \mathbf{v}_i^{ref} is the corresponding inertial frame reference vector, and index i runs over all sensors available at a given time. The matrix A is the “single-frame” attitude estimate at that time. Similarly, the attitude-dependent alignment estimation problem can be cast in a parallel form, that is: to solve for the misalignment for a given sensor by minimizing the loss function

$$L(O) = \sum_j \left[O \mathbf{v}_j^{body} - A_j \mathbf{v}_j^{ref} \right]^2 \quad (2)$$

as a function of the orthogonal misalignment O , where subscript j is a time index running over all valid observations, and A_j is the known attitude history. The vector \mathbf{v}_j^{body} is the observation unit vector for the given sensor, rotated to the body frame using the nominal alignment, N_o , and any a priori misalignment, M_o . Once O is determined, the new misalignment is usually expressed in the sensor frame as

$$M = N_o^{-1} O N_o M_o \quad (3)$$

where the inverse of N_o is used here rather than the matrix transpose to allow for possible nonorthogonality. It is clear Eqs. (1) and (2) can be minimized using the same method. Choosing the QUEST algorithm to solve Eq. (2) results in the ALIQUEST utility.

One possible complication is that one component of the TAM misalignment, O_{TAM} , may be poorly determined. This can be the case for spinning spacecraft where misalignment about the spin axis is not as observable as the other axes. This problem is resolved by solving also for the misalignment of the independent sensor, $O_{sensor-2}$. This independent sensor typically will be designed to have good sensitivity about the spin axis, such as with a Sun, Earth, or star sensor. To combine the misalignments into a single effective TAM misalignment, both sensors are rotated by the inverse of $O_{sensor-2}$

$$\begin{aligned} O_{TAM-combined} &= O_{sensor-2}^{-1} O_{TAM} \\ O_{sensor-2-combined} &= O_{sensor-2}^{-1} O_{sensor-2} = I \end{aligned} \quad (4)$$

Thus, the TAM alignment is defined relative to that of the independent second sensor, and the independent sensor is unmodified. This combined misalignment is computed and applied to the data on each iteration.

The method for determining the misalignment of the independent sensor depends on its type. ALIQUEST is a good choice if the measurements cover a wide field of view (FOV) in the body frame. However, the errors can be large when the FOV is small or when the measurement falls in the same bin on each spin period as it may with a single-head Sun sensor or slit star tracker. In this case, the software solves for a misalignment about the spin-axis using a very direct method. It projects the TAM and independent sensor observations and their reference vectors onto the plane perpendicular to the nominal spin axis and finds the mean rotation angles from the references to the observations. These mean rotations are the misalignment angles about the spin-axis. The difference between the TAM and independent sensor misalignment angles gives the relative error used to correct the TAM similarly to Eq. 4.

On each iteration, a new attitude history is determined and a new combined alignment is computed. The iteration is stopped when the change to the combined alignment is smaller than a given tolerance.

III. Test Results

A. Test 1 – ST5 Simulation

The calibration algorithm was tested with a series of simulations using parameters based on the ST5 mission. This mission consists of three small spinning spacecraft that were successfully launched together on March 22, 2006. The three share a Sun-synchronous orbit with an eccentricity of 0.24 and a period of about 137 minutes. The spacecraft spin at roughly 20 revolutions per minute (rpm). The sensors used for ground attitude determination are a single-head Sun sensor and a high resolution TAM, which also is the primary science instrument. Once per spin, the Sun sensor measures the angle from the body Z-axis to the Sun vector and generates a timing pulse at the body frame azimuth of the sensor slit. The TAM measurement frequency is 8 Hz, which gives about 24 observations per spin period. The TAM is deployed after launch on a 1-meter boom so a significant misalignment is not unexpected.

In the simulations, the spacecraft was assumed to be axially symmetric so an exact attitude history could easily be generated. The diagonal elements of the inertia tensor were taken to be [0.8; 0.8; 1.12] kg-m² with zeros off the diagonal. The sensor noise was modeled as white and zero-mean, with Sun sensor error of 0.18 deg and TAM error of 2 mG (corresponding roughly to 0.5 deg error). The simulations did not include maneuvers, but the initial angular momentum vector was offset from the body Z-axis by a 2 deg nutation angle. The simulations ran for one hour, starting 25 minutes before perigee.

For the most severe ST5 test, the TAM data was corrupted with errors in all the modeled parameters. The bias, scale factor, skew, and alignment errors are given in the upper part of Table 1. (The nonorthogonal skew angles are defined so that, e.g., the -1.5 deg X-axis skew represents rotation about the X-axis of the Y-axis towards Z. This also equals the skew angle of the Z-axis towards Y since the nonorthogonality is constructed symmetrically. Thus, in this case, the TAM axes have a Y-to-Z angle of 87 deg rather than 90 deg.)

The lower part of Table 1 shows the calibration results. The convergence tolerance was set to 0.04 deg for the orthogonal alignment; the results show good agreement with the truth model to this accuracy.

Table 1. TAM Calibration Truth Model Parameter Errors and Estimation Results for ST5 Simulation.

		TAM Bias (mG)	Scale Factor Error	Nonorthogonal Skew (deg)	Orthogonal Misalignment (deg)
Truth model	X-axis	-1	0.05	-1.5	1
	Y-axis	2	-0.05	1	3
	Z-axis	3	0.03	2	-2
Estimated Values	X-axis	-0.982	0.0496	-1.506	0.989
	Y-axis	1.993	-0.0500	0.986	3.014
	Z-axis	2.974	0.0303	2.013	-1.976

As a result of the calibration, the attitude errors are significantly reduced. The pre-calibration attitude error, relative to the truth model, has a standard deviation of 1.8, 1.9, and 0.7 deg on the X-, Y-, and Z-axes, respectively. (The first two minutes of the run are discarded from the statistics to allow for filter convergence.) The root-sum-square (RSS) of these three errors is 2.7 deg. The post-calibration RSS error is only 0.14 deg. This improvement is evident in Figures 1 and 2, which show the pre- and post-calibration attitude errors as differences from the truth model attitude. The pre-calibration systematic errors appear in Figure 1 as offsets from zero and large amplitude oscillations at the spin period due to error in the estimated spin-axis pointing direction.

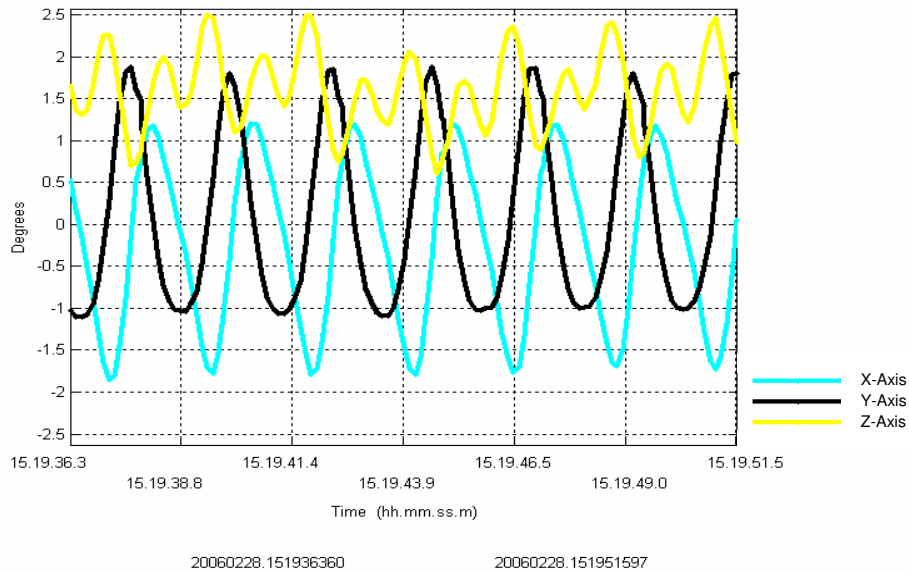


Fig. 1. ST5 simulation: Pre-calibration difference between truth model and attitude estimated using TAM and Sun sensor. Plot shows a typical 15 sec span from middle of 1 hour data set. Blue, black, and yellow curves show attitude errors as rotation angles about the body X-, Y-, and Z-axes, respectively.

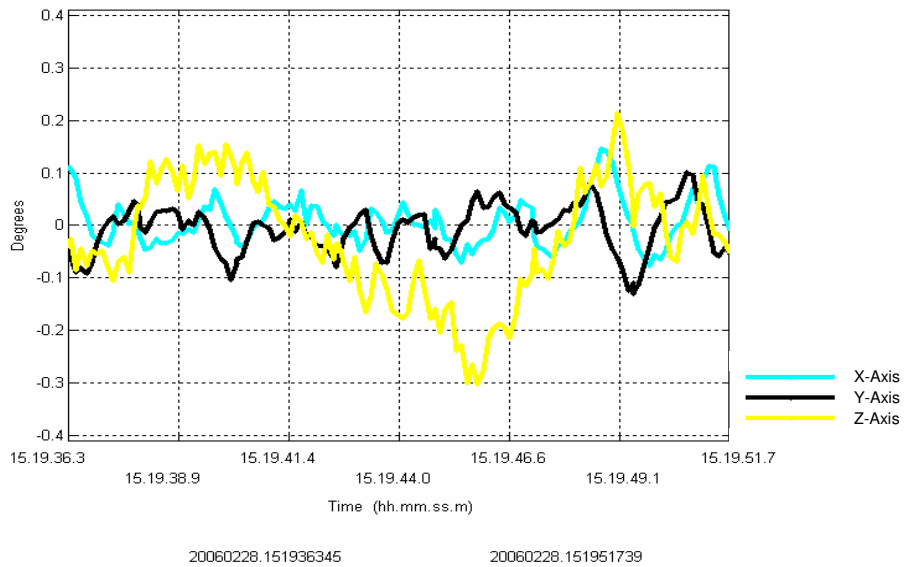


Fig. 2. ST5 simulation: Post-calibration difference between truth model and attitude estimated using TAM and Sun sensor. Plot shows a typical 15 sec span from middle of 1 hour data set. Blue, black, and yellow curves show attitude errors as rotation angles about the body X-, Y-, and Z-axes, respectively.

B. Test 2 – ST5 Flight Data

After the ST5 launch, NASA/GSFC provided some early data sets for analysis. At the time of this writing, there were still questions about the correct time-tags for the TAM data. An incorrect time bias on the TAM measurements relative to the Sun pulse each spin period would appear as a misalignment of the TAM about the spin axis (the Z-axis). Conversely, the TAM alignment calibration may help resolve the time bias problem. (Results given here are for the spacecraft designated #155.)

The ST5 data had been pre-processed to remove erroneous multiple Sun pulses and to establish TAM time-tags that were at least approximately correct. There remains a timing error equal to an unknown multiple of the spin period, but this is not observable here. The best available data was a 27-minute span starting 20 minutes before apogee.

Figure 3 shows the pre-calibration TAM residuals from the SpinKF estimator for a typical 15 sec time span from the middle of the data set. (The residuals are expressed approximately as angles in the body frame by dividing the differences between the observations and reference vectors by the magnitude of the field). The RSS of the standard deviations of the errors on the three axes is 1.73 deg for the TAM and 0.21 deg for the Sun sensor (not shown). As demonstrated in the previous section with simulated ST5 data, Figure 3 shows the signature of systematic errors in the TAM parameters and misalignment relative to the Sun sensor.

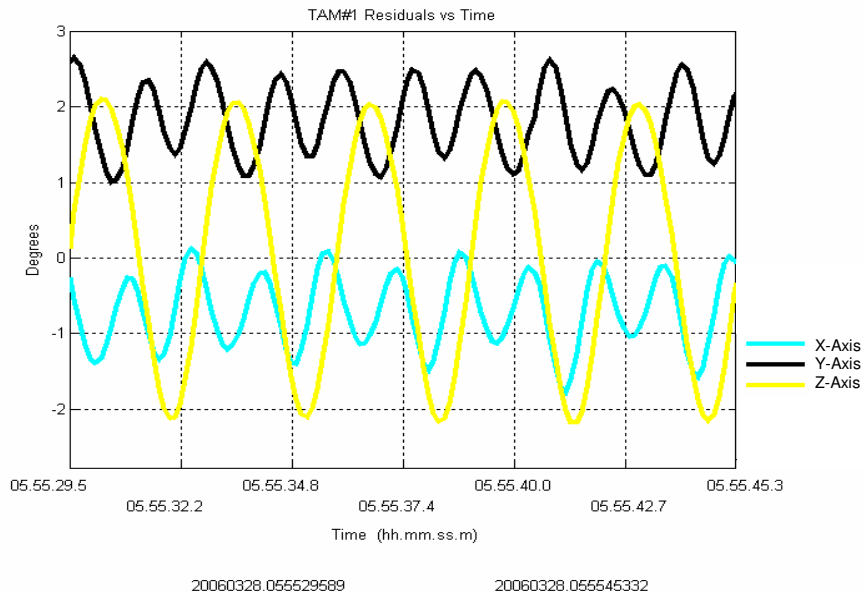


Fig. 3. ST5 Flight Data: Pre-calibration TAM residuals from the SpinKF attitude estimator expressed in degrees. Plot shows a typical 15 sec span from middle of data set.

Iterative TAM calibration for the ST5 flight data yields biases of -3.13, 1.53, and -0.01 mG, an orthogonal misalignment of 1.97, -0.49, and -0.31 deg, scale factor errors of 0.017, 0.019, and 0.012, and nonorthogonal skew angles of -0.25, -0.05, and -0.09 deg, on the X-, Y-, and Z-axes, respectively. This indicates that the TAM pre-processing had taken out most of the Z-axis bias and misalignment, but the X- and Y-biases and especially the X-misalignment are significant.

Figure 4 shows the post-calibration TAM residuals for the entire data set except for the first two minutes. The RSS errors have been reduced to 0.07 deg for the TAM and 0.14 deg for the Sun sensor. It appears that the random noise in the TAM is much smaller than the 2 mG assumed in the simulation tests.

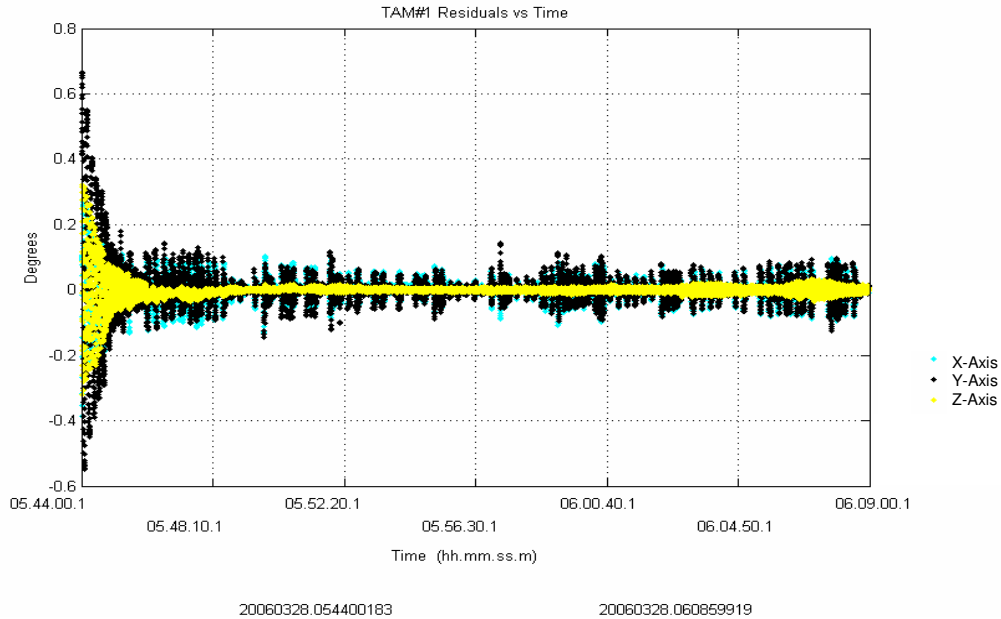


Fig. 4. ST5 Flight Data: Post-calibration TAM residuals from the SpinKF attitude estimator expressed in degrees. Plot shows entire data span except for the first two minutes of filter transients.

C. Test 3 – Aura Flight Data

The Earth Observing System (EOS) Aura mission is an Earth-oriented, three-axis stabilized spacecraft in a Sun-synchronous, 98 deg inclination, low-Earth orbit. Aura carries two star trackers and gyros for relatively high accuracy attitude determination and control. The attitude accuracy is 25 arcsec (3σ). The magnetometers are used as input for momentum management and for safe attitude control modes independent of the star trackers and gyros.

Mission personnel performed a series of careful TAM calibrations making use of data sets from the first six weeks of the mission.¹² The calibration method was a statistical minimization of a loss function constructed from TAM residuals. An independent, accurate, attitude history is needed for that method, and the onboard star tracker and gyro based attitude was used. The test described here compares the reported calibration results to those using the iterative method.

The Aura telemetry includes a flag indicating when the magnetic torquers are inactive. This allows the software to select only measurements when the torquers are off. However, the telemetry flagging was not perfect and additional manual screening of the data was necessary to remove all measurements showing any effects from the torquers. The data set included 14 orbits. Exactly an integer number of orbits was selected to average out any systematic orbit phase-dependent errors in the reference field model.

Figure 5 shows pre-calibration TAM residuals from the batch least-squares attitude estimator. The residuals here are body frame differences between observed and reference vectors in milliGauss. The figure shows only one orbit from the 14-orbit data set. The star tracker residuals for this attitude solution have a RSS standard deviation of 22 arcsec.

The pre-calibration TAM residual mean errors are -0.03, 19.73, and 19.74 mG, with standard deviations of 4.16, 2.84, and 8.23 mG for X, Y, and Z, respectively. The RSS of the mean errors is 27.91 mG and the RSS of the standard deviations is 9.65 mG.

Figure 6 presents the post-calibration TAM residuals. For clarity, only one component is plotted, and again, only one of the 14 orbits is shown. The post-calibration TAM residual mean errors are only -0.89, 0.13, and 0.15 mG, with standard deviations of 1.17, 1.42, and 1.49 mG for X, Y, and Z, respectively. The RSS of the mean errors is 0.91 mG and the RSS of the standard deviations is 2.37 mG.

Table 2 compares the new iterative calibration results with the statistical method results reported in Ref. 12 for the Aura mission. The values from Ref. 12 have been converted to the same frame and format as the iterative calibration parameters for comparison. Overall very good agreement between the methods is found.

Table 2. Aura TAM Calibration Results, Comparing the Operational Statistical Method and the Iterative Method.

		TAM Bias (mG)	Scale Factor Error	Nonorthogonal Skew (deg)	Orthogonal Misalignment (deg)
Statistical Method ¹²	X-axis	0.7	0.027	0.583	0.101
	Y-axis	21.0	0.006	-1.011	0.054
	Z-axis	20.3	0.030	1.816	-0.230
Iterative Method	X-axis	1.7	0.028	0.693	0.105
	Y-axis	20.7	0.005	-0.979	0.070
	Z-axis	20.1	0.034	1.813	-0.117

The TAM is given much less weight in the attitude estimator than the star trackers, so the systematic TAM errors do not affect the attitude solution. This is to be contrasted with the ST5 example where the TAM was a primary sensor and its errors had a large effect on the attitude accuracy. Also for this reason, the algorithm converged in two iterations for Aura, but required from five to eight iterations with various real and simulated ST5 data sets.

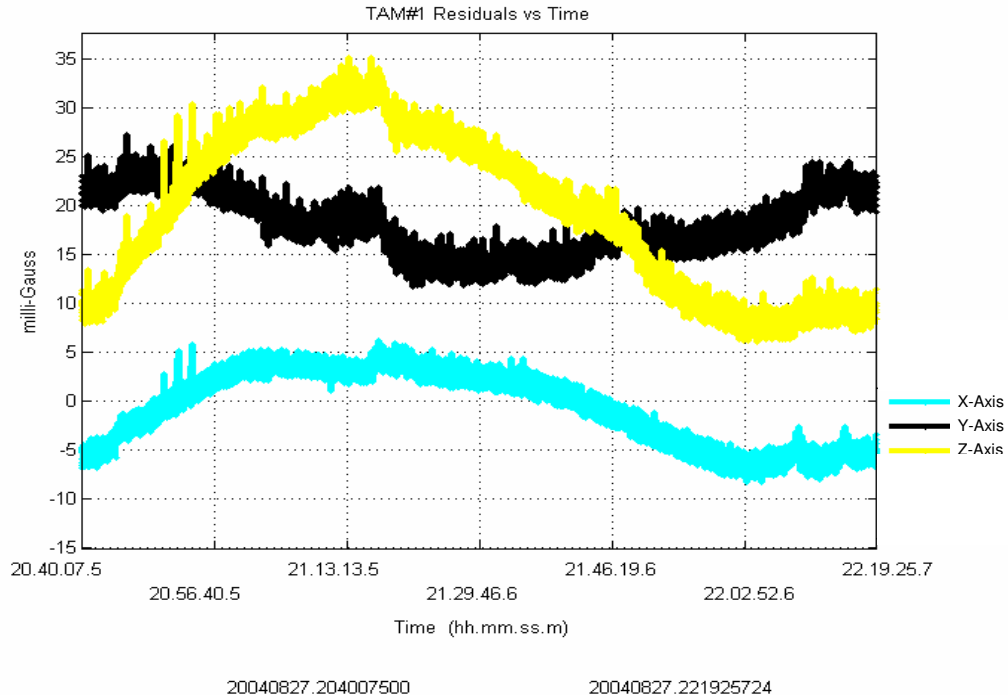


Fig. 5. Aura Flight Data: Pre-calibration TAM residuals (observation minus reference vectors) from the batch-least squares attitude estimator. Plot shows a typical orbit from the 14-orbit data set.

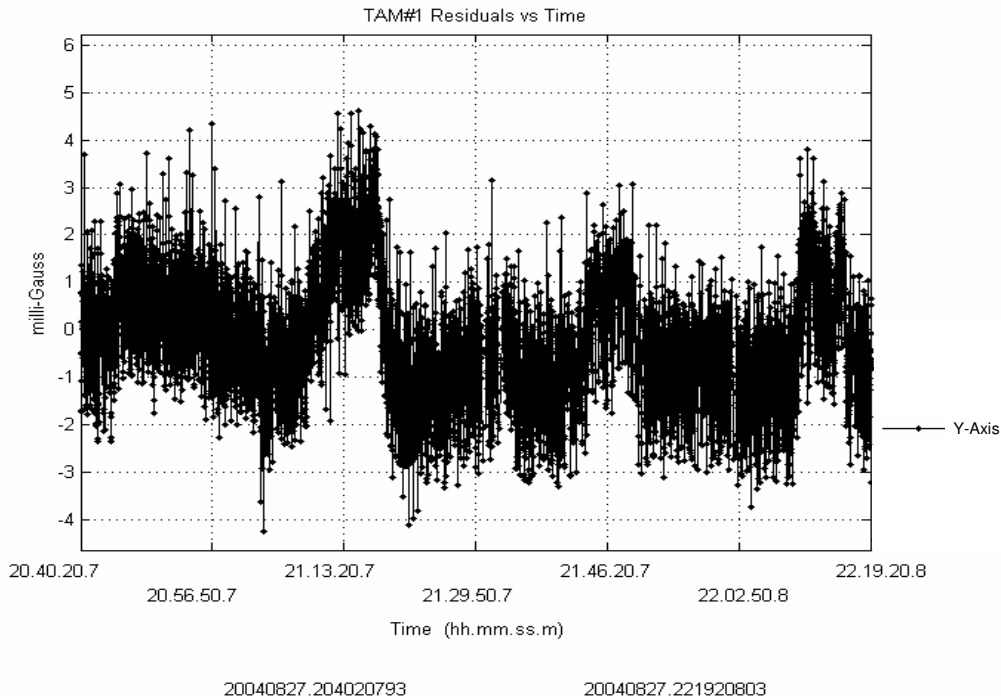


Fig. 6. Aura Flight Data: Post-calibration TAM residuals (observation minus reference vectors) from the batch-least squares attitude estimator. Plot shows a typical orbit from the 14-orbit data set. For clarity, only the Y-axis is shown; X and Z are similar.

IV. Conclusions

Tests with both simulated and actual flight data show that the iterative TAM calibration method provides accurate solutions for the full set of TAM calibration parameters, including biases, scale factors errors, nonorthogonality corrections, and orthogonal misalignment. Good results were obtained for both spin-stabilized spacecraft and Earth-oriented three-axis stabilized spacecraft.

More efficient methods could be devised other than unadorned iteration; however, there are good reasons for choosing this approach. The method is simple to implement, software had already been developed for each of the three steps, and it solves the problem of propagating sensor observations to a common time. In some form, propagation to a common time is necessary when solving for the relative alignment; information from different sensors must be compared at the same times. In the iterative method, this propagation is handled entirely by the attitude estimator. The state vector is propagated either with the state transition matrix (using gyro data) or by direct integration of the equations of motion. This is a great simplification for spinning spacecraft without rate sensors in that the integration does not need to be reproduced in the calibration software.

Alternative approaches that are under study may be able to improve the speed and efficiency of the calibration method. The attitude estimation step can be very slow, especially for the EKF and SpinKF since these filters must process the data sequentially. When three-axis gyro rate data is available, it would be simple to rotate neighboring observations to common times and compute an attitude history using a single-frame method. (Such a single-frame utility already exists in the ground support system – it is much faster than other attitude estimators.) The individual attitudes would have larger errors than those in a batch least-squares or EKF solution, but the mean error should be similar. When used in the attitude-dependent alignment estimation, the results may prove to be close to those obtained with the more accurate attitude estimators. When gyro data is not available, a different approach would be needed to improve the speed. Perhaps, one could solve for the attitude history once using SpinKF and linearize about that solution. Then, it may be possible to minimize the sensor residuals as a function of calibration parameters and an attitude state correction. Finally, SpinKF would be called again to recompute the attitude and residuals to validate the results. This could lead to faster convergence and fewer calls to the relatively slow attitude estimation step.

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